

# BULK SPIN-WAVE REGION IN AN ANTIFERROMAGNETIC SEMICONDUCTOR SUPERLATTICE

TANRIVERDIEV V.A., TAGIYEV V.S., SEID-RZAYEVA S.M.

*Institute of Physics, Azerbaijan National Academy of Sciences*

We present theoretical studies of superlattice formed from alternating layers of two simple-cubic antiferromagnetic semiconductors. The bulk spin-wave regions for spin waves propagating in a general direction in the superlattice are derived by the Green's function method. The results are illustrated numerically.

Keyword: magnetic superlattice

## 1 Introduction

In recent years much experimental and theoretical effort has gone into studies of magnetic multiplayer and superlattice (SL) structures. Various SLs have been prepared in which ferromagnetic and antiferromagnetic layers alternate. We can design the superlattices that we need with the aid of theoretical studies. These factors have aroused great interest in superlattice materials in recent years. It has been shown that multilayered systems have new, unique properties, not found in single-component systems [1-4]. However, a large number of problems of the theory of magnetic superlattices with the elementary unit cell consisting of different magnetic materials remain unsolved. Some qualitative features of superlattice are most easily explained for the simple - cubic structures in terms of modified single-film properties [5-7].

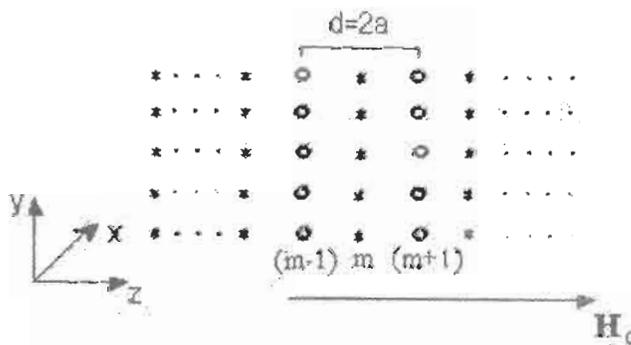


Fig. 1. Side view of superlattice model in which the atomic planes of material 1 alternate with atomic planes of material 2.

The properties of antiferromagnetic semiconductors superlattice comparatively fewer have been studied. There have been some calculations applying the s-f model to infinitely extended antiferromagnetic semiconductors [8-10]. One of the most interesting problems of magnetic semiconductors is related to the study of the electron-magnon interaction processes. In this paper, we study a simple cubic antiferromagnetic semiconductors superlattice mo-

del in which the atomic planes of material 1 alternate with atomic planes of material 2. Each atomic plane is assumed to be the [001] planes (fig 1). The exchange interaction between atoms of the two atomic layers at each interface is assumed to be antiferromagnetic but different from the corresponding bulk couplings. Dispersion equation of exchange spin waves for the superlattice under consideration is derived by the Green's function method [11,12].

## 2 Model Hamiltonian and equation of motion

The full Hamiltonian of the system is expressed as the sum of three terms: a Heisenberg Hamiltonian  $H_M$  for the localized spins (f type), a Hamiltonian  $H_B$  representing the kinetic

and Zeeman energy of the conduction (s) electrons, and s-f interaction Hamiltonian  $H_I$ , where

$$\begin{aligned} H_M &= \sum_{i,j} J_{ij} S_i S_j - \sum_i g\mu_B (H_0 + H_i^{(A)}) S_{i,a}^z - \sum_i g\mu_B (H_0 - H_i^{(A)}) S_{i,b}^z, \\ H_E &= \sum_{i,j} t_{ij} a_{i\sigma}^+ a_{j\sigma} - g_e \mu_B H_0 \sum_i s_i^z, \quad H_I = -\sum_i I_i S_i s_i. \end{aligned} \quad (1)$$

Here  $J_{ij}$  is the exchange integral between two nearest-neighbours spins,  $S_i$  and  $S_j$  are spin operators at the lattice sites  $i$  and  $j$ , respectively,  $a$  and  $b$  are number indexes of sublattices.  $H_0$  is the internal field applied in the  $z$  direction,  $H_i^{(A)}$  ( $i=1,2$ ) anisotropy field for an antiferromagnet with simple uniaxial anisotropy along the  $z$  axis. Also,  $t_{ij}$  is a hopping term and  $I_i$  is a contact interaction energy. The spin operators  $s_i$  of the conduction electrons at site  $i$  can be expressed as  $s_i^+ = a_{i+}^+ a_{i-}$ ,  $s_i^z = (a_{i+}^+ a_{i+} + a_{i-}^+ a_{i-})/2$ , where  $a_{i\sigma}^+$  and  $a_{i\sigma}$  are Fermi - creation and - annihilation operators at site  $i$ , respectively,  $\sigma = \pm 1$  corresponds to the spin - up and - down state.

To study of the magnetic excitations of the system we introduce four different types of Green's functions:

$$\begin{aligned} G_{i,j}(t) &= \langle\langle S_{i,a}^+(t) | S_{j,a}^-(0) \rangle\rangle, & F_{i,j}(t) &= \langle\langle S_{i,b}^+(t) | S_{j,a}^-(0) \rangle\rangle, \\ G'_{i,j}(t) &= \langle\langle s_{i,a}^+(t) | S_{j,a}^-(0) \rangle\rangle, & F'_{i,j}(t) &= \langle\langle s_{i,b}^+(t) | S_{j,a}^-(0) \rangle\rangle. \end{aligned} \quad (2)$$

Using the equation of motions for the Fourier transform of the Green's functions (2) in the RPA one obtain the following combined set of equations for the Green's function  $G_{i,j}(\omega)$  and  $F_{i,j}(\omega)$ :

$$\begin{cases} \lambda_i^a(\omega) G_{i,j}(\omega) - \langle S_{i,a}^z \rangle \sum_{\delta} I_{i,i+\delta} F_{i,j}(\omega) = 2 \langle S_{i,a}^z \rangle \delta_{i,j}, \\ \lambda_i^b(\omega) F_{i,j}(\omega) - \langle S_{i,b}^z \rangle \sum_{\delta} I_{i,i+\delta} G_{i,j}(\omega) = 0, \end{cases} \quad (3)$$

where

$$\lambda_i^{a(b)}(\omega) = \omega - g_e \mu_B (H_0 \pm H_i^{(A)}) + \sum_{\delta} J_{i,i+\delta} \langle S_{i+\delta,b(a)}^z \rangle - I_i \langle S_{i,a(b)}^z \rangle - \frac{I_i^2 \langle S_{i,a(b)}^z \rangle \langle S_{i,a(b)}^z \rangle}{\omega - g_e \mu_B H_0 - I_i \langle S_{i,a(b)}^z \rangle}$$

the upper sign refers to  $\lambda_i^a$  and lower one to  $\lambda_i^b$ ;  $\delta$  is the vector of location of the nearest neighbours.

Further more, to emphasize the layered structure we shall use the following Fourier transforms in the  $xy$  plane

$$G_{i,j}(t) = \frac{1}{N} \sum_{k_{\parallel}} G_{m,m'}(\omega, k_{\parallel}) \exp[ik_{\parallel}(r_i - r_j)], \quad F_{i,j}(t) = \frac{1}{N} \sum_{k_{\parallel}} F_{m,m'}(\omega, k_{\parallel}) \exp[ik_{\parallel}(r_i - r_j)]. \quad (4)$$

Here  $k_{\parallel} = (k_x, k_y)$  is a two dimensional wave vector parallel to the xy plane,  $m$  and  $m'$  denote the z components of the positions of the spins  $r_i$  and  $r_j$ , i.e. indices of the layers to which  $r_i$  and  $r_j$  belong, respectively. The normalization constant  $N$  denotes the number of sites in any of the lattice planes. Assuming that  $m$ -th layer is of the material 1 and  $(m+1)$ -th layer is of the material 2, one obtains the following set of equations

$$\begin{cases} \lambda_1^a(\omega)G_{m,m'} - 4J_1\langle S_1^z \rangle(I - \gamma(k_{\parallel}))F_{m,m'} - J\langle S_1^z \rangle F_{m-1,m'} - J\langle S_1^z \rangle F_{m+1,m'} = 2\langle S_1^z \rangle \delta_{m,m'} , \\ \lambda_1^b(\omega)F_{m,m'} + 4J_1\langle S_1^z \rangle(I - \gamma(k_{\parallel}))G_{m,m'} - J\langle S_1^z \rangle G_{m-1,m'} - J\langle S_1^z \rangle G_{m+1,m'} = 0 , \\ \lambda_2^a(\omega)G_{m+1,m'} - 4J_2\langle S_2^z \rangle(I - \gamma(k_{\parallel}))F_{m+1,m'} - J\langle S_2^z \rangle F_{m+2,m'} - J\langle S_2^z \rangle F_{m,m'} = 2\langle S_2^z \rangle \delta_{m+1,m'} , \\ \lambda_2^b(\omega)F_{m+1,m'} + 4J_2\langle S_2^z \rangle(I - \gamma(k_{\parallel}))G_{m+1,m'} + J\langle S_2^z \rangle G_{m+2,m'} + J\langle S_2^z \rangle G_{m,m'} = 0 , \end{cases} \quad (5)$$

where

$$\lambda_{1(2)}^{a(b)}(\omega) = \omega - g_1\mu_B(H_0 \pm H_{1(2)}^{(A)}) \mp 4J_{1(2)}\langle S_{1(2)}^z \rangle \mp 2J\langle S_{2(1)}^z \rangle \mp I_{1(2)}\langle s_{1(2)}^z \rangle - g_e\mu_B H_0 \mp I_{1(2)}\langle S_{1(2)}^z \rangle - I_{1(2)}^2\langle S_{1(2)}^z \rangle \langle s_{1(2)}^z \rangle / \omega$$

Here, the upper sign refers to  $\lambda_{1(2)}^a$  and lower one to  $\lambda_{1(2)}^b$ . Equation (5) is valid in the low temperature limit and random phase – approximation (RPA)  $\langle S_{1(2),a}^z \rangle = -\langle S_{1(2),b}^z \rangle = \langle S_{1(2)}^z \rangle$ ;  $\langle s_{1(2),a}^z \rangle = -\langle s_{1(2),b}^z \rangle = \langle s_{1(2)}^z \rangle$  have already been done.

The system is also periodic in the z direction, which lattice constant is  $d=2a$ . According to Bloch's theorem we introduces the following plane waves [13,14]:

$$\begin{aligned} G_{m+2,(m+1),m'}(\omega, k_{\parallel}) &= G_{m,(m-1),m'}(\omega, k_{\parallel}) \exp[ik_z d], \\ F_{m+2,(m+1),m'}(\omega, k_{\parallel}) &= F_{m,(m-1),m'}(\omega, k_{\parallel}) \exp[ik_z d] \end{aligned} \quad (6)$$

The set of equation (5) may be rewritten under following matrix form

$$Mu = w \quad (7)$$

$$M = \begin{pmatrix} \lambda_1^a & -4J_1\langle S_1^z \rangle(I - \gamma(k_{\parallel})) & 0 & -J\langle S_1^z \rangle[1 + \exp(-ik_z d)] \\ 4J_1\langle S_1^z \rangle(I - \gamma(k_{\parallel})) & \lambda_1^b & J\langle S_1^z \rangle[1 + \exp(-ik_z d)] & 0 \\ 0 & -J\langle S_2^z \rangle[1 + \exp(ik_z d)] & \lambda_2^a & -4J_2\langle S_2^z \rangle(I - \gamma(k_{\parallel})) \\ J\langle S_2^z \rangle[1 + \exp(ik_z d)] & 0 & 4J_2\langle S_2^z \rangle(I - \gamma(k_{\parallel})) & \lambda_2^b \end{pmatrix}$$

$$u = \begin{pmatrix} G_{m,m'}(\omega, k_{\parallel}) \\ F_{m,m'}(\omega, k_{\parallel}) \\ G_{m+1,m'}(\omega, k_{\parallel}) \\ F_{m+1,m'}(\omega, k_{\parallel}) \end{pmatrix}, \quad w = \begin{pmatrix} 2\langle S_1^z \rangle \delta_{m,m'} \\ 0 \\ 2\langle S_2^z \rangle \delta_{m+1,m'} \\ 0 \end{pmatrix}$$

The dispersion equation of spin waves propagating in a general direction in the superlattice under consideration is easily obtained by solving the set of equation (7).

$$\begin{aligned}
 & 256J_1^2J_2^2\langle S_1^z \rangle^2\langle S_2^z \rangle^2(1-\gamma(k_{\parallel}))^4 + 16(1-\gamma(k_{\parallel}))^2[\lambda_1^a(\omega)\lambda_1^b(\omega)J_2^2\langle S_2^z \rangle^2 + \lambda_2^a\lambda_2^bJ_1^2\langle S_1^z \rangle^2 - \\
 & - 4J_1J_2J^2\langle S_1^z \rangle^2\langle S_2^z \rangle^2(1+\cos(k_zd))] + \lambda_1^a(\omega)\lambda_1^b(\omega)\lambda_2^a(\omega)\lambda_2^b(\omega) + \\
 & + 2J^2(\lambda_1^a(\omega)\lambda_2^b(\omega) + \lambda_2^a(\omega)\lambda_1^b(\omega))\langle S_1^z \rangle\langle S_2^z \rangle(1+\cos(k_zd)) + 4J^4\langle S_1^z \rangle^2\langle S_2^z \rangle^2(1+\cos(k_zd))^2 = 0.
 \end{aligned}
 \tag{8}$$

### 3 Numerical results and discussion

Equation (8) is the main result of this paper. It can be verified from equation (8) that when both media are antiferromagnetic dielectrics  $I_1 = I_2 = 0$ , the equation (8) reduces to the well-known expression of bulk-spin wave dispersion equation for antiferromagnetic dielectric superlattice [14]. In fig. 2 a,b the results numerically illustrated for a particular choice of parameters. Fig. 2 b shows the spin-wave regions for the superlattice as a function of the quantity  $\gamma(k_{\parallel})$ , while fig. 2 a shows those for the components 1 and 2. Fig. 2 a and 2 b correspond to  $-1 \leq \cos k_z a \leq 1$  and to  $-1 \leq \cos k_z d \leq 1$  respectively.

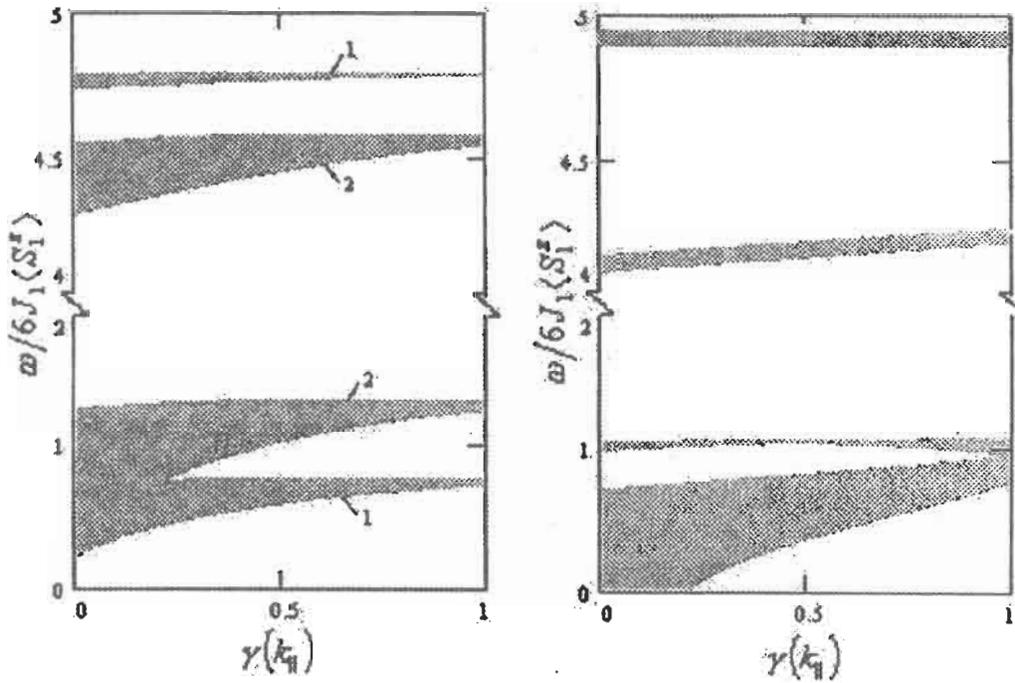


Fig.2. The spin wave frequencies (in unit of  $6J_1\langle S_1^z \rangle$ ) plotted against  $\gamma(k_{\parallel})$  in the low- and high-frequency regions. The parameter values are  $g_1\mu_B H_0/J_1\langle S_1^z \rangle = 0.3$ ,  $g_2 = g_e = 0.8g_1$ ,  $g_2\mu_B H_2^{(A)}/J_1\langle S_1^z \rangle = 0.18$ ,  $\langle S_1^z \rangle = \langle S_2^z \rangle = 3/2$ ,  $\langle s_{1(2)}^z \rangle = 1/2$ ,  $J_2/J_1 = 2$ ,  $I_1/J_1 = 20$ ,  $I_2/J_1 = 17$ . a) the bulk spin wave regions for constituents 1 and 2; b) the bulk spin wave regions in the superlattice for  $J/J_1 = 2$ .

The analysis of the results show that the width of the bulk-spin wave regions in the antiferromagnetic semiconductors superlattice is depended on transverse components of wave

vectors, s-f interaction and exchange interaction between localized spins. The bulk-spin wave regions appear in the low – frequency and in the high – frequency region. The bulk – spin wave regions move up, but the width of them decreases with increasing s-f interaction.

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**TANRIVERDİYEY V.Ə., TAĞIYEV V.S., SEYİD-RZAYEVA S.M.**

**ANTİFERROMAQNİT YARIMKEÇİRİCİ İFRAT QƏFƏSDƏ  
HƏCM SPİN DALĞA ZONASI**

İki müxtəlif sadə-kubik Heyzenberq antiferromaqnit atom layların növbələşməsindən alınan antiferromaqnit yarımkeçirici ifrat qəfəsdə həcm spin həyacanlamaları tədqiq edilir. Qrın funksiyası metodu ilə, ifrat qəfəsdə həcmi spin dalğalarının yayılmasını xarakterizə edən həcmi spin dalğaları zonası müəyyən edilmişdir. Alınmış nəticələr kəmiyyətcə təsvir edilir.

**ОБЛАСТИ ОБЪЕМНЫХ СПИНОВЫХ ВОЛН В АНТИФЕРРОМАГНИТНЫХ  
ПОЛУПРОВОДНИКОВЫХ СВЕРХРЕШЕТКАХ**

**ТАНРИВЕРДИЕВ В.А., ТАГИЕВ В.С., СЕИД-РЗАЕВА С.М.**

В настоящей работе рассмотрено распространение объемных спиновых возбуждений в антиферромагнитных полупроводниковых сверхрешетках, состоящих из чередующихся одноатомных слоев гейзенберговских кубических антиферромагнетиков двух типов. Методом функций Грина определены области объемных спиновых волн описывающие распространение объемных спиновых волн в сверхрешетках. Полученные результаты численно интерпретированы.